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Finite Gluons Ladders and Hadronic Collisions [*]

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A truncated BFKL series is studied and applied to hadronic processes. The $\sigma_{tot}^{pp(p\bar{p})}$ are described with good agreement with data and in a way consistent with the unitarity bound. The elastic scattering amplitude is calculated at $t \neq 0$, introducing two distinct ansätze for the proton impact factor. The $d\sigma^{el}/dt$ is obtained at small t approximation and compared with the data.

1. Introduction

The understanding of the BFKL Pomeron has been demanding a considerable theoretical effort. Its behavior in perturbative QCD is determined by generating the integral equation [1]. That procedure consists of summing the leading logarithms on energy, $\ln(s)$, order by order from perturbation theory and the main result is that the σ_{tot} for the $I\!\!P$-exchange process is a power of s.

A priori, BFKL is itself asymptotic and we may ask if at finite energies, i.e. non asymptotic regime, summing a finite number of terms from the BFKL series could describe the existent data. For that we should use a truncated BFKL series, performing a finite sum of gluon ladders (bearing in mind reggeized gluons and considering effective vertices). The question that remains is how many orders should one take into account. The lowest order two gluons exchange calculation leads to a σ_{tot} constant on s and the next contribution to the sum is the one rung gluon ladder, which provides a logarithmic growth. In order to avoid unitarity violation and by simplicity we truncate our summation at this order. As a result a successfull fit to the $\sigma_{tot}^{pp(p\bar{p})}$ with these two contributions is obtained and presented in Sec. 2. These results motivate to check the non-forward amplitude in order to obtain the prediction for the $d\sigma^{el}/dt$, which gives the behavior on the momentum transfer t.

In the BFKL framework such analysis is dependent of the proton impact factor (IF) input, which introduces certain uncertainty due to the presence of non-perturbative content. The IFs determine the coupling of the Pomeron to the color singlet hadrons and play a crucial role in the calculation of the non-forward amplitude. We calculate the $pp(p\bar{p})$ elastic amplitude at $t \neq 0$ taking into account two distict ansätze to the proton IF: the Dirac form factor, as proposed recently by Balitsky and Kuchina [2] and the usual non-perturbative ansatz [3]. The main resulting features are discussed in Sec. 3, although considering that a more realistic ansatz to the proton IF is still to be found. In addition the $d\sigma^{el}/dt$ is calculated in the small t approximation and compared with the experimental data. In the last section we present our conclusions.

2. Truncated BFKL Series

In the leading logarithm approximation (LLA), the Pomeron is obtained considering the color singlet ladder diagrams whose vertical lines are reggeized gluons coupled to the rungs through the effective vertices. The correspondent amplitude is purely imaginary and the coupling constant α_s is considered frozen in some transverse momentum scale [1]. For the elastic scattering of a hadron, the Mellin transform of the scattering amplitude is given by [4]:

$$\mathcal{A}(\omega, t) = \frac{\mathcal{G}}{(2\pi)^2} \int d^2\mathbf{k_1}\, d^2\mathbf{k_2}\, \frac{\Phi(\mathbf{k_1})\Phi(\mathbf{k_2})}{\mathbf{k_2}^2(\mathbf{k_1}-\mathbf{q})^2}$$
$$\times f(\omega, \mathbf{k_1}, \mathbf{k_2}, \mathbf{q}), \qquad (1)$$

where the $\mathcal{G}$ is the color factor for the color singlet exchange, $\mathbf{k_1}$ and $\mathbf{k_2}$ are the transverse momenta of the exchanged gluons in the t-channel and $\mathbf{q}$ is the momentum transfer, with $\mathbf{q}^2 = -t$.

The function $f(\omega, \mathbf{k_1}, \mathbf{k_2}, \mathbf{q})$ is the Mellin transform of the BFKL kernel $F(s, \mathbf{k_1}, \mathbf{k_2}, \mathbf{q})$, which states the dynamics of the process and is obtained in perturbative QCD. The main properties of the LO kernel are well known [1] and the results arising from the NLO calculations have yielded intense debate in the literature recently [5].

In the case of $pp(p\bar{p})$ scattering, the factor $\Phi(\mathbf{k})$ is the proton IF, which in the absence of a perturbative scale has a non-perturbative feature and furnishes the $I\!\!P$-proton coupling. This factor turns the amplitude infrared safe when the colliding particles are colorless. In the leading order of perturbation theory we have

$$f_1(\omega, \mathbf{k_1}, \mathbf{k_2}, \mathbf{q}) = \frac{1}{\omega}\,\delta^2(\mathbf{k_1}-\mathbf{k_2}), \qquad (2)$$

and in the next order

$$f_2(\omega, \mathbf{k_1}, \mathbf{k_2}, \mathbf{q}) = -\frac{\bar{\alpha}_s}{2\pi}\frac{1}{\omega^2}\left[\frac{\mathbf{q}^2}{\mathbf{k_1}^2(\mathbf{k_2}-\mathbf{q})^2}\right.$$
$$\left. -\frac{1}{2}\frac{1}{(\mathbf{k_1}-\mathbf{k_2})^2}\left(1+\frac{\mathbf{k_2}^2(\mathbf{k_1}-\mathbf{q})^2}{\mathbf{k_1}^2(\mathbf{k_2}-\mathbf{q})^2}\right)\right]. \qquad (3)$$

Here, $\bar{\alpha}_s = N_c\alpha_s/\pi$, where N_c is the color number and α_s is the strong coupling constant. In order to perform a reliable calculation the convenient proton IF should be introduced. This is not an easy task, namely these hadronic processes are soft and there is no hard scale allowing to use perturbation theory. In fact, we should know in details the parton wavefunction in the hadron to calculate the IFs properly. Since this is not available, several models are proposed in order to calculate them.

From the Optical Theorem, the lowest order contribution to the σ_{tot} is a constant term, and the next order term is a logarithm of the energy, scaled by a typical gluon transverse momentum of

the process (arbitrary). When considering $t = 0$ there is no need to deal with both a specific form for the IF and the transverse momentum integration. This allows to consider s-independent factors in each term as free parameters and to obtain them from data. The correct description at low energy requires the reggeon contribution, which is parameterized from Regge theory. Our expression to the total cross section is then,

$$\sigma_{tot}^{pp(p\bar{p})} = C_R(\frac{s}{s_0})^{\alpha_R(0)-1} + C_{Born} + C_{HO}\ln(\frac{s}{s_0}).$$

Hence we fix the constants C_{Born} and C_{HO} from data on $p\bar{p}$, imposing the same contribution for both pp and $p\bar{p}$. This procedure is reasonable due to the higher energies reached on $p\bar{p}$ collision, where the Pomeron dominates. On the other hand, pp data are predominantly at low energy, which is not strongly sensitive to the Pomeron model, thus dominated by the reggeonic contribution. A successful description of data is obtained for the whole range of energy. The result is shown in the Fig. (1). The parameters and a more detailed discussion can be found in Ref. [6].

The hypothesis of considering two orders from the BFKL series ($\sigma_{tot} \sim \ln(s)$) is phenomenologically corroborated by the well known dispersion relation fit [7]. An important additional advantage is that the total cross section obtained is consistent with the unitarity constraint, avoiding unitarization procedures. As a final remark, at the LHC energy ($\sqrt{s} = 14$ TeV) the extrapolation of our results will give $\sigma_{tot} = 93.22\ mb$.

3. The non forward scattering amplitude

In order to calculate the elastic amplitude at $t \neq 0$, information about the coupling between the proton and the t-channel reggeized gluons in the ladder is required. Namely, introducing a reliable proton IF that has to be modeled since it cannot be calculated from first principles due to the unknowledge on the wavefunction of the hadronic constituent partons. Here are analized two distinct models for the IF:
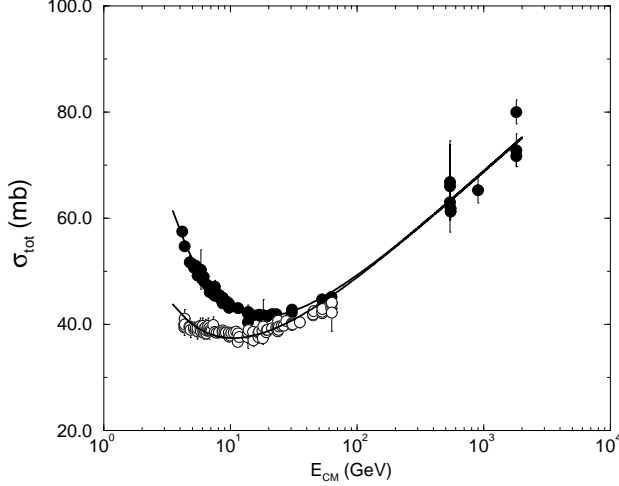
Figure 1. Result of the $pp(p\bar{p})$ total cross sections [8]. The errors are added into quadrature.

3.1. Dirac form factor:

Balitsky and Kuchina proposed recently [2] that at large momentum transfer the coupling of the BFKL Pomeron to the nucleon is essentially equal to the Dirac form factor of the nucleon. Their basic idea is that in the lowest order in perturbation theory there is no difference between the diagrams for the nucleon IF and similar diagrams with two gluons replaced by two photons, in such a way that the amplitudes can be calculated without any model assumption.

This IF, $\Phi_p(\mathbf{k}, \mathbf{q})$, is decoupled in the transverse momentum integration and presents an explicit dependence on t, being similar to the usual Pomeron-proton coupling used in Regge phenomenology. The expression is

$$F_1^{p+n}(t) = \frac{1}{1 + \left(\frac{|t|}{0.71\,GeV^2}\right)^2} \frac{4m_p^2 + 0.88|t|}{4m_p^2 + |t|} . \quad (4)$$

The choice for this proton IF is useful when one analyzes near forward observables, for instance

the elastic differential cross section. However it does not play the role of a regulator of infrared divergences at $pp(p\bar{p})$ process because clearly it does not vanish when the gluon transverse momenta goes to zero. In electron-proton process the situation is different since the photon impact factor supplies that condition [2].

Then the next step is to perform the gluon transverse momenta integrations. In fact, such integrals are infrared divergent and should be regularized. An usual way out is to introduce an infrared cut-off λ^2, temporally defining a small gluon mass, avoiding problems at the infrared region. This procedure is quite similar as to take into account a non-perturbative massive gluon propagator (see i.e. Ref. [9]).

The lowest order (order α_s^2) contribution, using Eqs. (1-2), gives the following result:

$$\mathcal{A}^{(1)}(s,t) = \frac{\mathcal{G}'}{(2\pi)^4} s \left[F_1^{p+n}(t)\right]^2 \frac{\pi}{(|t| - \lambda^2)} \ln\left(\frac{\lambda^2}{|t|}\right) .$$

The one rung gluon ladder has two components (order α_s^3), given by the following expression:

$$\mathcal{A}^{(2)}(s,t) = \frac{\mathcal{G}'}{(2\pi)^4} \left[F_1^{p+n}(t)\right]^2 s \ln\left(\frac{s}{\mathbf{k^2}}\right) (I_1 + I_2),$$

with I_1 corresponding to the one rung gluon ladder and I_2 correspondent to the three gluons exchange graphs, whose order is also $\ln(s/\mathbf{k^2})$. Such structure is due to the fact that in the color singlet calculation there is no cancellation between graphs and one can not obtain an expression for the two-loop level which is proportional to the one loop amplitude [4]. We define I_2 through symmetry on the integration variables $\mathbf{k_1}$ and $\mathbf{k_2}$ (see Eqs. (1,3)) and the factor $\mathcal{G}'$ collects the correspondent color factors and the remaining constants. The explicit calculation of those integrals, yields

$$I_1 = -\pi^2 \frac{|t|}{(|t| - \lambda^2)^2} \ln^2\left(\frac{\lambda^2}{|t|}\right) ,$$

$$I_2 = \frac{1}{2} \frac{\pi^2 \ln(\lambda^2)}{(|t| - \lambda^2)} \ln\left(\frac{\lambda^2}{|t|}\right) \left(1 - \frac{\ln(|t|)}{\ln(\lambda^2)}\right) .$$

Some comments about the amplitude above are in order. The scale of the factor λ^2 should be at

non-perturbative regime, i.e. < 1 GeV^2. An interesting aspect is the behavior of the amplitude at the forward limit $t = 0$, where it becames very large. This limit is a well known property of perturbative QCD calculations and there are several reasons to believe that the point $t = 0$ plays a very special role, such that perturbation theory may even not be applicable. For the full BFKL series in the forward region there is still the diffusion on tranverse momenta, i.e. on $\ln \mathbf{k}^2$, which extends into both the ultraviolet and the infrared regions [4]. Nevertheless, the momentum scale t supplies the control condition.

However, we will suppose that a smooth transition from a finite t down to $t = 0$ is possible and that the finite BFKL series gives the correct behavior on energy for the forward observables. Later we make use of this hypothesis to obtain the logarithmic slope $B(s)$ and the differential elastic cross section.

3.2. Usual non-perturbative ansatz:

Using quite general properties of the IFs, namely they vanish as transverse momenta go to zero, one can guess their behavior which is determined by the large scale nucleon dynamics. Regardless its exact shape, in general the proton IF takes the form [3]:

$$\Phi_p(\mathbf{k}) = \frac{\mathbf{k}^2}{\mathbf{k}^2 + \mu^2} , \qquad (5)$$

where μ^2 is a scale which is typical of the non-perturbative dynamics. As a consequence of this choice the momentum transfer behavior is completely determined by the kernel. The amplitude now reads:

$$\mathcal{A}(s,t) = A' \left[\frac{1}{(|t| - \mu^2)} + \frac{|t|}{(|t| - \mu^2)^2} \ln\left(\frac{\mu^2}{|t|}\right) \right]$$
$$+ A' \ln\left(\frac{s}{\mathbf{k}^2}\right) \left[\frac{\ln(\mu^2)}{(|t| - \mu^2)} + \frac{\ln(\mu^2)|t|}{(|t| - \mu^2)^2} \ln\left(\frac{\mu^2}{|t|}\right) \right] .$$

Here we use the definition $A' = \frac{\mathcal{G}}{(2\pi)^4} s \pi$.

We observe again a divergent behavior at $t = 0$, nevertheless we claim that the forward amplitude is finite in this point and the dependence on energy is correctly described. Despite obtaining an

analytic expression to the elastic scattering amplitude, a direct comparison with the experimental data is known not to be reliable. In fact, data on differential cross section at low t are parameterized in the form $d\sigma/dt = A e^{Bt}$, where B is the forward slope [7]. Therefore, we can obtain an expression for the differential cross section at small t, using our previous results.

The usual relation to describe the cross section is:

$$\frac{d\sigma^{el}}{dt} = \frac{d\sigma}{dt}|_{t=0} e^{B(s,\, t=0)\, t} = \frac{\sigma_{tot}^2}{16\pi} e^{B_{el}(s)\, t} , \quad (6)$$

$$B(s) = \frac{d}{dt} \left[\log \frac{d\sigma}{dt} \right]_{t=0} . \qquad (7)$$

In the Regge framework the slope is obtained from the powerlike behavior of the scattering amplitude, dependent of the effective slope of the Pomeron trajectory α'_P, namely $B_{el}^{Regge}(s) = 4b_0 + 2\alpha'_P \ln(s)$. The parameter b_0 comes from the slope of the p-p-$I\!P$ vertex. In our case we should obtain the slope from the non forward elastic scattering amplitudes $\mathcal{A}^{Ladder}(s,t)$ obtained above. For the amplitude obtained employing the Balitsky and Kuchina impact factor it results the following slope

$$B(s) = \frac{4}{F_1^{p+n}(t)} \frac{dF_1^{p+n}(t)}{dt} + \frac{2}{\mathcal{A}(s,t)} \frac{d\mathcal{A}(s,t)}{dt} |_{t=0},$$

where the first term does not contribute effectively at $t = 0$ and we are left only with the second term. From simple inspection of the amplitude obtained with the usual impact factor (see Eq. (5)) we also verify that one gets a similar expression to the correspondent slope.

Considering the specific form for the t-derivative of the amplitudes, their asymptotic values at $t = 0$ depend only on the energy. In fact, they take the form $d\mathcal{A}/dt = R_1 s + R_2 s \ln(s/s_0)$, where R_1 and R_2 are s-independent parameters. For our case, the amplitude is purely imaginary, then $|\mathcal{A}(s, t = 0)| = s \sigma_{tot}$ and $d\sigma/dt |_{t=0} = \sigma_{tot}^2/16\pi$. Putting all together, the corresponding slope is

$$B(s) = \frac{2}{\sigma_{tot}} [R_1 + R_2 \ln(s/s_0)] , \qquad (8)$$

and the elastic differential cross section is given by Eq. (6), where again $s_0 = 1\, GeV^2$.

In order to obtain the parameters R_1 and R_2, we use the slope experimental values for both low (CERN-ISR) and high energy (CERN-SPS,Tevatron) points from $p\bar{p}$ reaction (23 GeV$<$ $\sqrt{s} <$ 1800 GeV) [8]. Having the slope obtained from data, the elastic differential cross section is straighforwardly determined and a successful comparison with its experimental measurements at $\sqrt{s}$ = 1800 GeV is shown in the Fig. (2). In summary, we study the contribution of a trun-
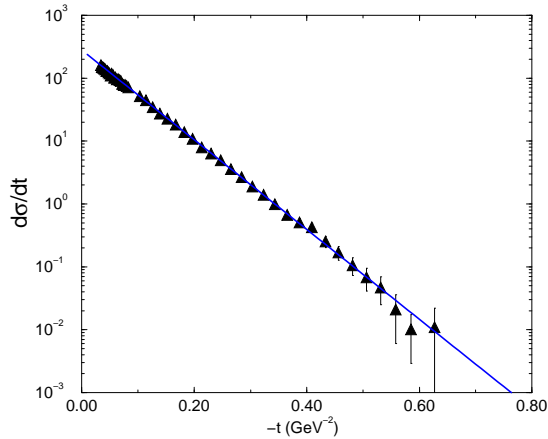

Figure 2. The result for the elastic differential cross section at $\sqrt{s} = 1800$ GeV [8].

cated BFKL series to the hadronic process, specifically the $pp(p\bar{p})$ collisions, considering two orders in perturbation theory corresponding to the bare two gluons exchange and the one rung gluon ladder. Despite the restrictions imposed by the use of a perturbative approach for soft observables, a good description of the total cross sections was obtained motivating an analysis of the elastic differential cross section. Although the QCD perturbation theory is in principle not reliable at the forward direction ($t = 0$), nevertheless we suppose that perturbation theory gives the behavior on energy even in this region. The next step is to consider t different from zero, where the momentum transfer furnishes a scale to perform suitable calculations. In order to proceed this, we calculate the non forward amplitude introducing two distinct ansätze for the proton impact factor, namely a factorizable t-dependent proposed recently by Balitsky and Kuchina and the usual non-perturbative impact factor. In order to describe data we used a small momentum transfer approximation and obtained an expression to the elastic slope $B_{el}(s)$ and the correspondent parameters. The elastic differential cross section is obtained straightforwardly, describing with good agreement the experimental data at both low and high energy values.

Acknowledgements

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